Structural reliability analysis of a marquee element

Análise de fiabilidade estrutural de um elemento de marquise

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ABSTRACT
Usually, projects of torsion beams situations caused by marquees are made considering a prescriptive methodology, not considering the unsureness related to the problem. When such uncertainties are ignored, projects can be oversized or not satisfactorily safe. An option to overcome this problem is an application of reliability theory, which consists of quantifying structural safety using probability of failure. Thus, the probability of failure of a beam subjected to torsion caused by a marquee is estimated in this article. For this, the reliability methods FORM (First Order Reliability Method) and Monte Carlo are applied. The reliability analysis are evaluated considering the variation of the geometric

dimensions and the properties of the materials involved. It is also done an analysis of the sensitivity indexes of the random variables that are part of the proposed problem limit state equation. The random variable of greatest contribution to failure was the specific weight of the concrete, with 60% of the composition of the probability of failure. The Fck, in turn, was the random variable that contributed the most to the reliability of the beam. The results demonstrate the importance of considering the uncertainties inherent in structural project of beams subjected to torsion.

Keywords: Structural analysis, Structural element, Probability of failure, FORM, Monte Carlo, Sensitivity Index, Marquee, Precast Concrete.

RESUMO
Normalmente, os projectos de situações de feixes de torção causados por marquises são feitos considerando uma metodologia prescritiva, não considerando a falta de segurança relacionada com o problema. Quando tais incertezas são ignoradas, os projectos podem ser sobredimensionados ou não ser satisfatoriamente seguros. Uma opção para ultrapassar este problema é uma aplicação da teoria da fiabilidade, que consiste em quantificar a segurança estrutural utilizando a probabilidade de falha. Assim, a probabilidade de falha de uma viga sujeita a torção causada por uma marquise é estimada neste artigo. Para tal, são aplicados os métodos de fiabilidade FORM (First Order Reliability Method) e Monte Carlo. A análise da fiabilidade é avaliada considerando a variação das dimensões geométricas e as propriedades dos materiais envolvidos. É também feita uma análise dos índices de sensibilidade das variáveis aleatórias que fazem parte da equação de estado limite do problema proposto. A variável aleatória de maior contribuição para o fracasso foi o peso específico do betão, com 60% da composição da probabilidade de fracasso. A Fck, por sua vez, foi a variável aleatória que mais contribuiu para a fiabilidade da viga. Os resultados demonstram a importância de considerar as incertezas inerentes ao projecto estrutural das vigas sujeitas a torção.

Palavras-chave: Análise estrutural, Elemento estrutural, Probabilidade de falha, FORMA, Monte Carlo, Índice de Sensibilidade, Marquee, Betão Pré-fabricado.

1 INTRODUCTION

Torsion is one of the most complex stresses that can be applied to structural elements. The most common case of torsion in structures occurs with cantilever slabs, embedded in support beams, such as marquees.

Traditionally, designs of torsion beams caused by marquees are made according to a purely prescriptive (deterministic) methodology, which does not directly consider the uncertainties related to the problem.

In this methodology, coefficients of increase and reduction are applied to the equations of limit state in an attempt to guarantee structural security. However, the use of such coefficients, besides being able to oversize the structure, do not allow estimating the level of security to which users will be subject (BAILEY, 2006) and (BECK, 2014).

Alternatively, the reliability theory can be applied to consider the uncertainties inherent to the problem.

Beck (2014) defines reliability as the subjective probability that a system will not fail and, probability of failure, the subjective probability that there will be a system failure when it does not meet the design specifications. Thus, in general, the reliability theory is given by an arrangement of equations responsible for quantifying the safety of the object of study.

The reliability theory, according to Beck (2014), can be applied through approximate and exact methods, being the approximate: FOSM (*First Order Second Moment)*; FORM *(First Order Reliability Method)*; SORM *(Second Order Reliability Method)*. For the exact method, we have the Monte Carlo simulation method.

For the preparation of this article, it was generally chosen the FORM method and the Monte Carlo method. The FORM method has been chosen due to its low computational cost and efficiency to estimate the probability of failure. In addition, it makes it possible to calculate the sensitivity indices. The Monte Carlo method has been chosen to validate the results obtained by the FORM method.

In this context, this article applies the reliability theory to determine the probability of failure of a beam subjected to torsion, caused by a marquee. In addition, sensitivity indices are determined via FORM to verify the percentage of contribution of each random variable in the composition of the probability of failure.

2 THEORETICAL FOUNDATION

In engineering, structural reliability analysis is commonly applied in order to estimate the possibilities of a particular structural system failing to fulfill its specific function. The satisfactory operating conditions are given according to the equation of limit state, which in turn are constituted by a set of variables, which are uncertainties described by random variables, commonly described by probability distributions and their own parameters.

Random variables are mathematically defined as a real function $\mathrm{X}(\omega)$ that assigns to each sample point ω of a sample space Ω a real value x, such that the set $\{X \leq x\}$ be an event for any real number x, (Beck, 2014). When the distributions are made up of several random variables, joint distributions are said, so that the RVs are represented by means of vectors $X = \{X_1, X_2, X_3, \dots, X_n\}$.

The random variables, depending on the structural model adopted, may contribute to the failure domain of the limit state equation, thus representing the failure of the structure. The violation of a limit state will result in an undesirable condition of the structure, that is, the ultimate limit state (ELU, loss of resistant capacity, progressive collapse and fatigue of the structure) and the service limit state (ELS, damage to the aesthetic appearance, comfort and durability of the structure), NBR 6118:2014. The limit state equation *g* is represented below, where R describes the random variable according to the resistance parameters, S the random variable according to the structural load., (*n*-*m*) and (*1*-*k*) the number of random variables that constitute the resistance and structural load variable, respectively:

$$g(X) = R(X_1, X_2, \dots, X_k) - S(X_m, \dots, X_n) \quad \text{Eq.1}$$

For values of $g \leq 0$, it is said to have exceeded the limit state, that is, failure, and $g > 0$, the domain of survival of the structure. Based on this definition, it is possible to divide domain *D* into failure domain D_f and survival domain D_s, (Beck, 2014):

$$D_f = \{X | g(X) \leq 0\} \quad \text{Eq. 2}$$

$$D_s = \{X | g(X) > 0\} \quad \text{Eq. 3}$$

Problems in general that have *n* variables, with their respective random variables, under the rule of previously defined limit state equations, can have their failure probabilities calculated on the failure region by approximating the integration of the following probability density function together with the use of an indicator function I [X]:

$$P_f = \int_{D_f} f_X(x)dx \quad \text{Eq. 4}$$

Among the methods usually applied in the scope of computational structural reliability, the FORM stands out as a method of approximation and Monte Carlo simulation as an exact method that strives to determine the solution to the integral in a direct way.

The FORM method uses information from the random variables that make up the limit state equation, it is approximated to a linear function and the equation's integration domain is defined approximately. The accuracy of the FORM method will depend on the configuration of the failure domain in the standard normal space.

The FORM method also provides a sensitivity index, that is, the relative percentage of contribution of each random variable to the occurrence of the failure domain of the limit state equation. This contribution is subject to the moments, the types of distribution of each random variable, and its alocation in the limit state equation. There are several types of sensitivity measures, such as importance factors, omission factors or metric sensitivity factors. Below is the importance factor for random variables, (Beck, 2014):

$$I_i = \alpha_i^2 \quad \text{Eq. 5}$$

Where α_i is the cosine director, which in turn can be algebraically described by:

$$\alpha_i = -\frac{\nabla g(y^*)}{||\nabla g(y^*)||} \quad \text{Eq. 6}$$

Where $\nabla g(y^*)$ is the gradient of the limit state equation at the design point y^*.

The Monte Carlo method (MCM), in turn, is a simulation method, which, based on a number of hypotheses and models, represents the real case. It is considered as an exact method because, theoretically, considering an infinite number of simulations, its result tends to an exact solution, (Beck, 2014). The disadvantage of MCM is evident in problems that have a low probability of failure, so that a relatively larger number of interactions are necessary for the method to point out reliable probability of failure (Ching, 2011). Therefore, it will require great computational cost.

As one of the theoretical tools for constituting the limit state equation for the problem to be presented in this work, the torsion equations of element of non-circular section were made available. Calling *L* the length of the bar, *a* and *b* the measure of the longest and shortest side, respectively, of its cross section, and *T* the intensity of the torsional moments applied to the bar, we have that the maximum shear stress occurs along the centerline of the longest side of the bar, being equal to:

$$\tau_{máx} = \frac{T}{c_1 \times a \times b^2} \quad \text{Eq. 7}$$

The coefficient c_1 depends only on the ratio *a/b* and is tabulated for a set of values of this ratio (Beer et al., 2011, p. 207).

In addition to the torsion equations of elements of non-circular section, the composition of the limit state equation also used the shear stress equation in beams and for calculating the transverse reinforcement strength and shear strength of the section without transverse reinforcement, NBR 6118:2014 (Brazilian Association of Technical Standards, ABNT), Design of Structural Concrete – Procedures, has been consulted.

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3 RESEARCH METHODOLOGY

In order to analyze the sensitivity index and the probability of failure of a marquee subjected to twisting, the structure shown in Figure 1 and Figure 2 was considered:

Figure 1: Marquee subjected to a load *q*, side view.

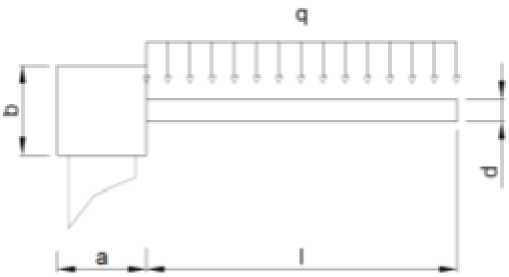

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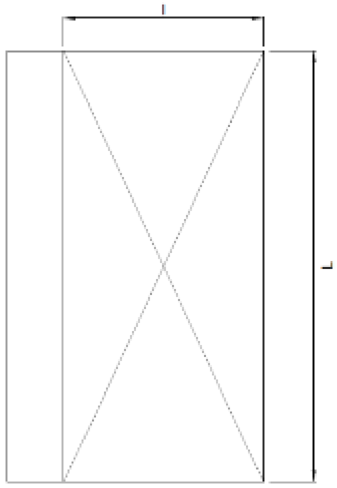
Figure 2: Marquee, topview.



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From the initial configuration presented above and the static equilibrium analysis and normative considerations of NBR 6118 (2014), the limit state function formulated below was obtained:

$$g(X) = 0,126 \cdot fck^{2/3} + 0,0054 \cdot fck^{2/3} - \left[\frac{1}{4} \cdot (l \cdot L \cdot d \cdot \gamma + Cv \cdot L \cdot l) \cdot (a + l) \cdot \frac{1}{c_1 \cdot a \cdot b^2} + \frac{3 \cdot L \cdot l \cdot (d \cdot \gamma + Cv)}{4 \cdot a \cdot b} \right] \quad \text{Eq. 8}$$

Where:

L, l, a, b, d, are the dimensions of the marquee pieces, represented in a drawing;

c_1, coefficient for rectangular bars in torsion;

C_v, average variable wind load;

γ, specific weight of reinforced concrete;

f_{ck}, characteristic compressive strength of concrete.

The variables included in the aforementioned limit state equation Cv, γ and f_{ck} are ruled by different types of random variables. In this study, wind was considered as a variable load Cv acting on the structure of the marquee. According to Brazilian normative recommendations in NBR 6120:1980 (*Loads for the calculation of building structures),* the average variable load to be applied on ceilings without access to people, situation of the marquee slab of the proposed problem, is 0.5 KN / m^2. According to Santiago (2011), the probability distribution of the variable request is Gumbel, with a standard deviation of 0.125 KN / m^2. Based on NBR 6118: 2003, if the specific mass of reinforced concrete is unknown, the estimate of this should be made by adding 100 kg/m^3 to 150 kg/m^3 to the specific mass of simple concrete already known. The concrete of normal specific mass, after going through an oven drying process, has a specific mass ρ_c between 2000 kg/m^3 and 2800 kg/m^3. In this work, it was considered, under gravitational acceleration of approximately 10 m/s^2, the specific weight of reinforced concrete γ having normal distribution, with an average of 25 KN/m^3 and standard deviation of 3 KN/m^3.

The average value for the average compressive strength of concrete f_{cm} is given from the characteristic compressive strength of concrete, the f_{ck}, according to NBR 12655: 2006 (*Concrete, preparation, control and receiving)*. To calculate the f_{cm}, the equation shown below is used:

$$f_{cm} = f_{ck} + 1{,}65 \times \sigma \qquad \text{Eq. 9}$$

The value of standard deviation σ equal to 4 MPa for high quality concrete, produced in concrete, must be admitted. For medium quality works, with the concrete produced in loco, σ of 5.5 Mpa is allowed (Botelho & Marchetti, 2010). It is important to highlight that reinforced concrete marquees must, at least, according to NBR 6118: 2003, be classified in the Aggressiveness Class CAA II (moderate aggressiveness with little risk of deterioration of the structure), according to table 6.1 of the referred standard. This implies adopting a concrete class C25 (fck = 25MP) or higher (Gonçalves, 2011).

Therefore, f_{ck} = 30 MPa was considered for the analysis of the problem proposed here and a standard deviation of 4 MPa, with normal distribution.

Table 1: Measures of the variables Cv, γ and fck

Variable	Average	Standard deviation	Unity	Type of distribution	Reference
C_v	0,5	0,125	kN/m^2	Gumbel	Santiago
γ	25	3	kN/m^3	Normal	Admitted by authors
f_{ck}	36,6	4	MPa	Normal	Botelho

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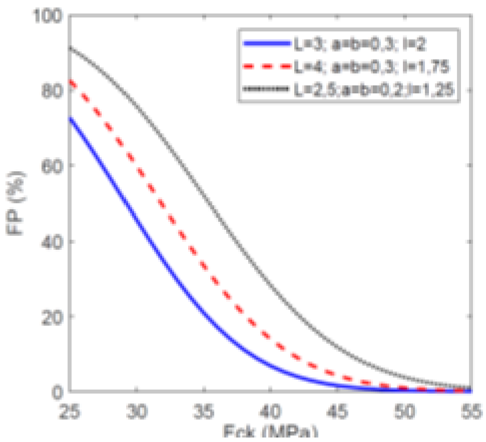
The structural failure analysis of the beam presented in this work was performed using the "Rt" (Risk Tools) software, made available by the Department of Civil Engineering at the University of British Columbia, Vancouver-Canada (MAHSULI & HAUKAAS, 2013).

4 RESULTS AND DISCUSSION

After the methodological application described above, the graphs were generated containing results for the analysis of the failure probability (FP) made from combinations of some parameters of the beam under torsion, which are displayed in the legend of Figure 3:

Figure 3: FP X Fck.



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The graphics are, according to the legend, configured by an array of parameters, which are the dimensions of the marquee: the length *L*, the square cross section of the beam with horizontal geometric characteristics *a* and vertical *b* and the width *l* of the slab section. Analyzing the continuous line graph of L = 3 m, it can be seen that, under the respective dimensions, there is a failure probability of 72.7%, for the marquee considering f_{ck} = 25MP. As expected, as the characteristic compressive strength of concrete increased, the FP was lower. Initially, the structural resistance reacted quite significantly, with greater reductions in its probability of failure per increased fck interval when compared to the final intervals shown in the graph. When the f_{ck} is raised from 25 to 30 MPa, the failure probability leaves its initial value of 72.7% and assumes 45.5%, a 27.2% drop on safety favor. As for the final variations from 50 to 55 MPa, the FP is reduced from 0.26% to 0.034%, a decrease of 0.226%.

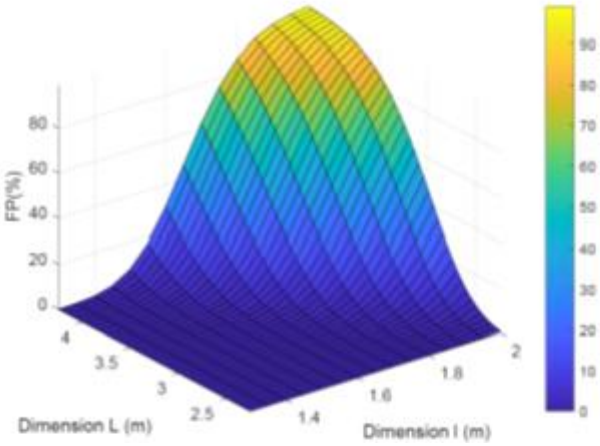
Checking the dashed graph it is possible to notice an increase of 9.88% of FP in comparison to the continuous line graph for f_{ck} of 25 Mpa. Although the dashed configuration has a smaller width for the slab than the continuous one, a reduction of 25 cm, this, in turn, has dimension L increased by 1 meter, this means an increase in the area of the slab, which makes it understandable to increase the probability, given the significant direct influence of geometric dimensions on the final result.

The dotted graph is arranged by geometric measures significantly smaller than the previous ones. The L dimension has been reduced 50% in relation to the same variable of the dashed curve. The significant reduction in the area of the slab logically leads to a lower performance of loads distributed along the beam, however the present graph presents, for fck = 25 Mpa, FP = 91.25%, an increase of 8.65% in relation to the dashed graph, or 18.55% in relation to the continuous line graph. A possible explanation for the significant decrease in beam reliability, since the marquee assumes smaller geometric measures falls on the cross section of the slab of dimensions *a* and *b*. By reducing it, the stress demands due to torsion and shear are consequently increased.

The curves maintain similar profiles throughout the fck variation. It is interesting noting that they do not intersect, as evident in the dotted curve, holding, within the analyzed range, higher probabilities of failure than the other graphs.

Another analysis has also been carried out after fixing the following values to the variables: square cross section with 30 cm edge, dimension d = 10 cm and f_{ck} = 30 MPa. Figure 4 illustrates the failure probabilities obtained via FORM method for the combinations of dimensions L (2.25m to 4.25m) and l (1.3m to 2m).

Figure 4: FP X (L,I).



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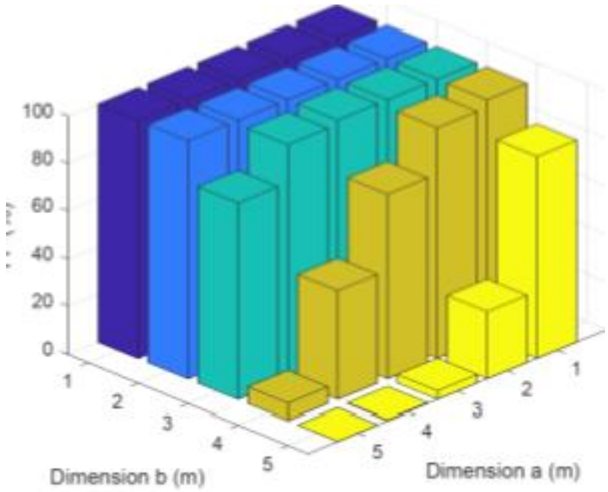
As the cross-section is kept constant, it is worth noting that the probabilities of failure increase as L and l are increased. Initially, for L = 1.25 m and l = 1.3 m the probability that the resistant capacity of the structure will be exceeded is 0%. Reliability remains constant and has a low growth trend for the first combinations between measurement intervals. Around L = 3.1 m and l = 1.7 m it was possible to verify 1.6% failure probability, although low, a significant minimum value. According to the graph, it is easy to verify that when adopting a low measure for one of the dimensions and a high value for another, the probability tends to remain small, as, for example, for L = 4.25 and l = 1.45 FP = 2.95%. Through this, it's worth noting the importance of the participation of both dimensions for the fault domain of the limit state equation.

From the proposed intervals, as expected, for the limit measures of L = 4.25 m and l = 2.0 m, there was a 99.17% failure probability. The possibility of an accident occurring becomes greater at an exponential rate for a given interval as increasing values are adopted for two-dimensional variables. As the final limit values of the measurements are approximated, there is a decrease in the rate of increase in the probabilities of failure. Probabilistic analyzes were also carried out when the dimensions of the beam cross section were varied, having been considered L = 3.0 m; l = 2.0 m; d = 10 cm; f_{ck} = 30 MPa, according to Figure 5:

Figure 5: FP X (a,b).



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The following measures are considered for dimensions *a* and *b* shown in a graph:

Table 2: Dimensions measurements *a* and *b*

Coordinate	Dimension (m)	
	A	**B**
1	0,15	0,15
2	0,2	0,2
3	0,25	0,25
4	0,3	0,3
5	0,35	0,35

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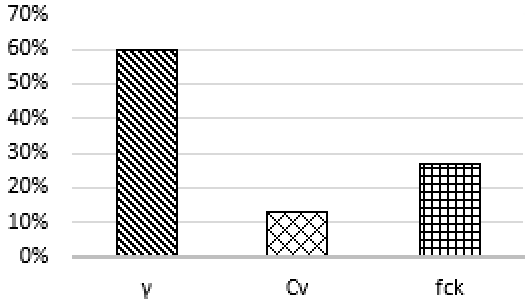
Contrary to the other analyzed dimensions (L, l), it may be seen that structural reliability is reduced as higher values for *a* and *b* are admitted. The safety of the structure proves to be very sensitive to variations in the cross section. Assuming *a* and *b*, 20 cm and 35 cm, respectively, a failure probability of 27.92% was obtained. Reducing 5 cm to the measure of a, 84.6% chance of ruin was calculated using the FORM method, an increase of about 300% in relation to the contiguous result. For a = b = 25 cm, FP = 99.97% was found. Considering maximum and minimum measures of the proposed interval, with a = 35 cm and b = 15 cm, 100% of failure was obtained. For a = 15 cm b = 35 cm, FP was 84.6%. Thus, it appears that dimension *b*, in relation to *a*, has a greater contribution to the occurrence of the failure domain.

An admissible interpretation for the growing trend of structural failure from the section reduction consists of equation 7, where the maximum shear stress is inversely proportional to the dimensional measurements of *a* and *b*.

Analyzing the structural reliability of the beam using the Monte Carlo method, considering the configuration shown in figure 3 (L = 3m; l = 2m; a = b = 30 cm; f_{ck} = 25MPa), a failure probability of 73.3% was obtained, resulting in a percentage difference of 0.6% in relation to the FORM method. Considering a case in figure 4 (L = 4.25 m; l = 1.45 m; a = b = 30 cm; d = 10 cm; fck = 30MPa), with a total number of samples of 69.749, it was found the result of 3.46% FP (Monte Carlo method), having been calculated 2.95% using the FORM method. In the analogical sequence, applying the MCM to one of the situations presented in figure 5 under the following configuration: L = 3.0 m; l = 2.0 m; a = 35 cm; b = 30 cm; d = 10 cm; f_{ck} = 30 MPa. FP = 8% has been obtained, the percentage difference being 0.94% in relation to that calculated by FORM.

Figure 6 shows the sensitivity indices of the random variables at the design point for the limit state equation, considering L = 3.0 m; l = 2.0 m; d = 10 cm; a = b = 30 cm; f_{ck} = 25 MPa. It is observed in figure 6 that the random variable that contributes the most to the composition of the failure probability is the specific weight of the concrete γ, 60%, followed by the fck and variable wind load, Cv, 27% and 13%, respectively. The contribution of the concrete specific weight is interpreted by the way in which the variable is presented in the limit state equation and its standard deviation, which does not present a large percentage difference in relation to those of the other variables.

Figure 6: Sensitivity index at the design point for the limit state equation.



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The characteristic compressive strength of concrete fck was the only random variable that contributed negatively to the occurrence of structural failure. This can be understood by considering that this variable acts in favor of the structural security of the element.

5 CONCLUSIONS

In this article, the failure probability in a beam subjected to torsion caused by a marquee was studied. For this, reliability methods (FORM and Monte Carlo) were applied to estimate the failure probability of the structural element and to calculate the sensitivity indices of the random variables. The results led to the following conclusions:

(i) The failure probability for the proposed beam model is highly sensitive to dimensional variations in its cross section. The reduction in the dimensions of the cross section results in an increase in the FP of the beam;

(ii) The results obtained by the analysis via FORM and Monte Carlo were similar, showing small percentage differences in their respective values when an analogy was made;

(iii) The sensitivity indices can serve as indicative for the designers since they point out the most critical random variables in the composition of the failure probability. Thus, the designer can concentrate efforts in order to decrease the contribution of the most critical variables and consequently increase the structural safety of the beam.

In general, the results demonstrate the importance of considering the uncertainties inherent in structural designs of beams under torsion, while they can serve as guidelines for designers to develop safer structures.

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