A literature review of dual hesitant fuzzy sets

Uma revisão da literatura sobre conjuntos dual hesitant fuzzy

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ABSTRACT
Group decision making is a common and crucial human activity, where two or more specialists handle complex decision problems, involving conflicting and non-measurable attributes. Thus, new decision-making approaches have been proposed in order to deal with different types of imprecisions. The Dual Hesitant Fuzzy Sets (DHFS) is a recent approach that stands out in the literature due to its ability to combine the advantages of Hesitant Fuzzy Sets and Intuitionistic Fuzzy Sets representations, allowing different types of hesitations by the decision makers. However, no paper has been identified that seeks to conduct a literature review on this theme. Therefore, the purpose of this paper is to present the results of a systematic literature review to assess the state of the art of DHFS. In addition, current applications are presented, and opportunities for future studies are elucidated.

Keywords: Dual Hesitant Fuzzy Sets, group decision, literature review.
RESUMO
A tomada de decisão de grupo é uma atividade humana comum e crucial, pela qual dois ou mais especialistas podem lidar com complexos problemas de decisão, envolvendo atributos conflitantes e não comensuráveis. Dessa forma novas abordagens de tomada de decisão vêm sendo propostas para lidar com diferentes tipos de imprecisões. O Dual Hesitant Fuzzy (DHF) é uma abordagem recente que se destaca na literatura devido à sua capacidade de combinar as vantagens das representações Hesitant Fuzzy e Intuitionistic Fuzzy, permitindo diferentes tipos de hesitações. Entretanto, não foi identificado nenhum artigo que busca realizar uma revisão da literatura sobre esta abordagem. Portanto, o objetivo deste artigo é apresentar os resultados de um levantamento bibliográfico sistemático para avaliar o estado da arte do DHF. Além disso, as aplicações existentes são apresentadas e oportunidade de estudos futuros são levantados.

Palavras-Chave: Conjuntos Dual hesitant fuzzy, decisão em grupo, revisão sistemática.

1 INTRODUCTION
The decision making is one of the most important functions of managers in any organization, due to its increasingly complexity and competitive environment [Nooraie 2012]. The decision-making process is a common activity in which techniques can be used to subsidize problems of selection, evaluation, ordering and categorization of available alternatives [Zhang et al. 2016]. Considering the evolution of production and management systems, such as the development of industry 4.0, one of the current and important challenges is the modeling of decision systems through consensus approaches involving uncertainty and fuzzy logic [Molano et al. 2017]. Thus, different techniques with different linguistic representations are proposed in the literature in order to deal with imprecision [Ghorabaee et al. 2017].

The theories based on fuzzy sets stand out in the applications, because they are the most used for data analysis with inaccuracies in strategic problems for the organization [Chang and Hung 2010]. The MCGDM (Multi-Criteria Group Decision Making) techniques proposed in the literature use specific fuzzy representations that aim to address the problem of group decision-making. Representations of information often found in the literature that are included into the mentioned category are: intuitionistic fuzzy sets [Ebrahimnejad et al. 2015], hesitant fuzzy sets [Gitinavard et al. 2016], interval-valued intuitionistic fuzzy sets [REN et al. 2017] and interval-valued hesitant fuzzy sets [Gitinavard et al. 2017]. A comparison of these fuzzy representations applied to supplier selection problem is presented in Calache (2018).

Although the techniques based on intuitionist fuzzy and hesitant fuzzy representations are able to model the uncertainties of the group decision-making process,
each representation has been used to deal with a specific type of imprecision of these problems. Hesitant fuzzy representations are more appropriate to be used when there are doubts within the judgment, where one or more linguistic terms can be used in the evaluation process [Zhang et al. 2016]. Intuitionist fuzzy representations are used when it is desired to add a margin of hesitation for definition of the fuzzy numbers that represent the linguistic terms [Nguyen 2016].

A new generalization of fuzzy sets was proposed by Zhu et al. (2012), the Dual Hesitant Fuzzy Sets (DHFS). This representation seeks to combine intuitionist fuzzy and hesitant fuzzy concepts, uniting the advantages of each. The hesitant fuzzy sets were proposed by Torra (2010), allowing the membership degree of an element in a set to assume several possible values, enabling the decision maker's hesitation in defining the variables that represent his judgment [Zhang et al. 2017]. As in fuzzy intuitionistic sets, DHFS also have membership and non-membership degree functions; however, these two functions are expressed by several fixed numbers, instead of a single one, modeling the imprecision of the real world more precisely than other fuzzy theory generalizations [Yu et al. 2016].

Recently, DHFS have been widely used for multicriteria decision-making problems, with the development of new models and theories [Zhang et al. 2017]. However, no study has been found presenting the state of the art of this approach. Thus, this study brings a systematic review that aims to elucidate the state of the art of Dual Hesitant Fuzzy Sets and present possibilities for future studies.

2 BASIC CONCEPTS OF DUAL HESITANT FUZZY SETS

Zhu et al. (2012) define the concept of dual hesitant fuzzy as an extension of hesitant fuzzy sets. Given a fixed set U, a Dual Hesitant Fuzzy Set (DHFS) is represented as in Eq. (1):

\[
\tilde{D} = \{(x, \bar{h}_D(x), \bar{g}_D(x)) \mid x \in U\}
\]

(1)

Where \(\bar{h}_D(x)\) and \(\bar{g}_D(x)\) are two sets of the range \([0,1]\), denoting the membership and non-membership degree of the element \(x \in U\) in the set \(D\), with the following conditions respectively: \(0 \leq \gamma, \eta \leq 1 \) and \(0 \leq \gamma^+ + \eta^+ \leq 1\), for all \(x \in U \gamma \in \bar{h}_D(x), \eta \in \bar{g}_D(x)\), \(\gamma^+ \in \bar{h}_D(x), \eta^+ \in \bar{g}_D(x) = \bigcup_{\gamma \in \bar{h}_D(x)} \text{Max}\{\gamma\}, \eta^+ \in \bar{g}_D(x) = \bigcup_{\eta \in \bar{g}_D(x)} \text{Max}\{\eta\}.\)
The basic operations and properties of DHFS sets were also presented by Zhu et al. (2012). Given three DHF elements, \( d, d_1 \) and \( d_2 \) and a non-negative integer number \( n \), then the basic operations can be presented as in equations (2) to (5) [Yu et al. 2016]:

Union- Sum: \( d_1 \oplus d_2 = \bigcup_{\bar{y}_1, \bar{y}_2 \in \tilde{h}_1, \tilde{y}_1, \tilde{y}_2 \in \tilde{g}} \{(\tilde{y}_1 + \bar{y}_2 - \tilde{y}_1 \bar{y}_2, \tilde{y}_1 \tilde{y}_2)\} \) (2)

Intersection-Multiplication: \( d_1 \otimes d_2 = \bigcup_{\bar{y}_1, \bar{y}_2 \in \tilde{h}_1, \tilde{y}_1, \tilde{y}_2 \in \tilde{g}} \{(\tilde{y}_1 \bar{y}_2, \tilde{y}_1 + \bar{y}_2 - \tilde{y}_1 \bar{y}_2)\} \) (3)

Multiplication by \( n \): \( nd = \bigcup_{\bar{y} \in \tilde{h}, \bar{y} \in \tilde{g}} \{(1 - (1 - \bar{y})^n, \{\bar{y}^n\})\} \) (4)

Potentiation by \( n \): \( d^n = \bigcup_{\bar{y} \in \tilde{h}, \bar{y} \in \tilde{g}} \{\bar{y}^n, \{1 - (1 - \bar{y})^n\}\} \) (5)

Let \( \tilde{d}_j = (\tilde{h}_j, \tilde{g}_j) (j = 1, 2, ..., n) \) be a set of DHFS elements, then the score function \( S(d_j) \) is defined as in Equation (6):

\[
S(d_j) = \frac{1}{\#h} \sum \gamma_j \tilde{h}_j - \frac{1}{\#g} \sum \eta_j \tilde{g}_j
\] (6)

For example, if \( S(d_3) > S(d_1) > S(d_2) \), then \( d_3 > d_1 > d_2 \).

Based on the concepts proposed by Zhu et al. (2012), Wang et al. (2014) developed some aggregation operators, among them the Dual Hesitant Fuzzy Weighted Average (DHFWA) and the Dual Hesitant Fuzzy Weighted Geometric (DHFWG) which are used to calculate weighted averages and weighted geometric averages of judgments based on DHFS [Zeng et al. 2018]. Regarding linguistic variables, as can be seen in studies using DHF linguistic variables, the representations of activated terms follow the same principle as the Hesitant Fuzzy linguistic representation. However, the DHFS linguistic representation also considers the imprecision that comes from the hesitation of parameterizing the linguistic terms, thus having membership degrees and non-membership degrees for the terms activated by the judgments [Wang et al. 2014]. The use of these representations allows decision makers to make more complex judgments through the use of sentences. Thus, Rodrigues et al. (2013) present a proposal for the transformation of sentences into hesitant linguistic terms sets.
3 RESEARCH METHOD

In order to present the state of art on dual hesitant fuzzy approaches and their applications, a survey was conducted through a systematic review. Systematic reviews differ from narrative and exploratory analyses, adopting a replicable, scientific and transparent process [Tranfield et al., 2003]. The research procedures are described below and follow the steps proposed by Sampaio and Mancini (2007):

- Analysis of the research question: in the present work, it was sought what are the applications of the DHFS approach presented in the literature, as well as to understand the state of the art and verify possibilities of advances in the subject;

- Definition of databases and keywords: Web of Science (apps.webofknowledge.com), SCOPUS (www.scopus.com) and EMERALD (https://www.emerald.com/insight/) databases were used due to their wide collections related to operations management subject. The set of words ("dual hesitant fuzzy") was applied, returning a total of 80 papers in the Web of Science database, 86 in SCOPUS and 10 in EMERALD.

- Search strategy: All papers from journals, congresses and books that returned from the search dated until the end of 2019 were analyzed. Duplicated papers were eliminated resulting in a total of 126 papers. The search strategy was first to read the abstract and keywords, and in case of doubt, the following sections of the papers were read to evaluate their scope.

- Selection and classification of relevant papers: 116 papers were selected that were directly related to the application or development of the DHFS approach. These papers were read and classified according to the year of publication, country of the first author's institution, application theme and multi-criteria technique combination.

4 RESULTS AND DISCUSSION

Since its proposition, the Dual Hesitant Fuzzy Sets has presented a rapid increase in the number of studies that have been developing the approach and/or applying it to different types of contexts. Figure 1 shows the evolution of the publications number over the years related to DHFS. Although the approach is recent in the literature, its origin is based on the combination of Intuitionistic Fuzzy and Hesitant Fuzzy generalizations, which are already widely applied in group decision-making problems [Qu et al. 2017].
Despite the increase in the number of publications related to the approach, according to Figure 2, it can be seen that the subject study is concentrated in a few countries. China stands out as the country with the largest number of papers with 82% of share, followed by India with 10% of contributions. No country in the Americas has been found having studies related to DHFS. To map these countries is important to identify possible partnerships for international collaborations.

Regarding the dissemination of content by journals, it is possible to verify a concentration of papers in journals focused on the development and application of fuzzy theories. The Journal of Intelligence & Fuzzy Systems presents a higher number of DHFS articles with 22.4% of the total contributions. However, 53 different journals have published articles on the subject, and considering the last years of 2019 and 2020, there is a wide dissemination of the subject in various high impact journals, such as Computers & Industrial Engineering, Information Sciences and Symmetry for example. This survey can
help authors to identify the most complacent journals on the subject, and thus direct their submissions.

Regarding the DHFS applications, the papers were classified as presented in Table 1. The literature presents proposals for applications of Dual Hesitant Fuzzy Sets representation for several management problems. Despite the variety of applications, there is still a small concern with aspects related to sustainable development, with only 4.3% of studies addressing related themes [Qi et al. 2018; Qu et al. 2017; Qi et al. 2016; Zhang et al. 2016]. Moreover, most studies are developing advances in the DHFS approach, such as the proposal of new aggregation operators (49%) [Liang et al. 2019; Zhang et al. 2019], methods of assessing the distance between evaluations (7%) [Liu et al. 2020; Chen et al. 2018], among others, as shown in the graph of Figure 3. Thus, the applications are, in general, illustrative for exemplifying the new propositions.

Table 1: Classification of DHFS studies applications.

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<thead>
<tr>
<th>Application themes</th>
<th>References</th>
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<td>Evaluation and/or selection of suppliers: articles that present problems related to the process of choosing a new supplier or evaluation of the performance of existing suppliers or services</td>
<td>Yu et al. 2016; Qu et al. 2017; Qu et al. 2016; Qi et al. 2018; Wei 2017; Li et al. 2019; Ren et al. 2017; Yu et al. 2016; Zhao et al. 2016; Zhang et al. 2018; Wei 2017; Wang et al. 2016.</td>
</tr>
<tr>
<td>Human resources: staffing; employee selection; performance evaluation.</td>
<td>Su et al. 2019; Yu 2014; Garg e Kaur 2019; Rodzi 2020; Bashir et al. 2019; Ren e Wei 2017; Yu e Li 2014; Singh 2014.</td>
</tr>
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It is important to note that, so far, few multicriteria techniques have been combined with the DHFS approach and few decisions making models have been proposed with the approach [Lu and Wei 2016; Wang et al. 2016; Xu 2016; Zhang 2016]. The consensus approaches developed so far are related to decreasing the distances between evaluations among decision makers, and for this purpose, optimization models have been proposed [Gong and Chen 2019; Maity et al. 2019]. Due to the complexity of the Dual Hesitant Fuzzy Sets based approaches, optimization models include multiple objectives and/or nonlinear programming [Qu et al. 2019; Li et al. 2019]. It was not found papers that combines metaheuristics with DHFS, which can bring less complexity in the resolution of these problems, although it does not guarantee the optimal solution [Ehrgott and Gandibleux 2008; Viana and De Souza 2000].

**Figure 3:** Techniques used in conjunction with DHFS.

5 CONCLUSIONS

This study presented the current state of the art regarding the Dual Hesitant Fuzzy approach. This form of information representation stands out in the literature due to its ability to deal with different types of imprecisions in information. Hence, the advantages of the hesitant fuzzy and intuitionistic fuzzy approaches have been combined, making it possible to model the hesitancy of the decision makers assessments and in the parameterization of the used linguistic variables.

Through the survey it was possible to observe that the technique presented a rapid increase in the number of studies and applications. A possible explanation is in its ability to deal with the group decision-making process. Thus, the vast majority of the works seek
to aggregate the judgments of decision makers, trying to model the different points of view within an organization [Tang et al. 2018; Qi et al. 2018; Yu et al. 2016]. On the other hand, some studies bring consensus techniques to minimize the divergence of the group of decision makers to reach a common solution [Zhang et al. 2016; Zhao et al. 2016]. However, studies dealing with consensus techniques do not mathematically process linguistic terms. Therefore, with the results of the literature review, it was possible to identify some possibilities for future studies, such as:

- The combination of problem structuring methods for elucidation of alternatives and criteria and application of the DHFS approach for evaluating the performance of alternatives;
- The combination of different multi-criteria techniques with DHFS representation to consider the different types of hesitation in a decision process;
- The combination of metaheuristics with DHFS representation to deal with group decision problems involving multiple objectives and/or presenting a nonlinear programming model;
- The development of a prescriptive model that can guide decision makers in the decision making process considering different steps and approaches for different specific objectives, such as problem structuring, consensus among experts and judgments aggregation;
- Applying the DHFS approach in real contexts, especially considering the context of sustainable development, which is scarcely explored in the studies raised in this review.

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